APPLICATION OF DISCRETE LIGHT-CONE QUANTIZATION TO YUKAWA THEORY IN FOUR DIMENSIONS[*]

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The numerical technique of discrete light-cone quantization (DLCQ) is applied to a single-fermion truncation of Yukawa theory in four dimensions. The truncated theory is regulated by three Pauli–Villars bosons, which are introduced directly in the DLCQ Fock-state basis. A special form of the Lanczos diagonalization algorithm is used to handle the indefinite metric. Renormalization is done nonperturbatively.

1. Introduction

Methods for the nonperturbative numerical solution of light-cone-quantized quantum field theories have progressed to the point where they are applicable in four dimensions. The use of light-cone coordinates[1] makes possible a meaningful Fock-state expansion, in which no disconnected vacuum contributions appear. The Fock-state wave functions in the expansions are obtained by solving a Hamiltonian eigenvalue problem.[2] A standard technique for solving such a problem is discrete light-cone quantization (DLCQ).[3,2] The wave functions are evaluated at discrete momentum values $p^+ \equiv E + p_z = n\pi/L$, $\vec{p}_\perp \equiv (p_x, p_y) = \vec{n}_\perp \pi/L_\perp$, with n, n_x, and n_y integers and L and $L_\perp$ length scales. The coupled integral equations that comprise the eigenvalue problem become a matrix diagonalization problem where trapezoidal sums approximate integrals.

To properly formulate such a matrix problem as an approximation to a four-dimensional field theory, one must include a regularization scheme and perhaps additional cutoffs that yield a finite matrix problem. One must also include a renormalization scheme to determine bare parameters. These steps have been carried out for simple models[4,5] and are now being applied to Yukawa theory.[6] The regularization scheme is based on the introduction of Pauli–Villars particles,[7] including some with negative norm, and of a simple mass counterterm. The theory is then finite before discretization. A cutoff on the light-cone energy $(m^2 + p_\perp^2)/p^+$ is used to limit the transverse momentum range and produce a matrix representation of finite size. Renormalization is done nonperturbatively, by fixing computable quantities to "data."

The original theory is recovered in the following sequence of limits. First, the

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numerical limit of infinite longitudinal and transverse resolutions is taken at fixed cutoff and fixed Pauli–Villars masses. Next, the cutoff is removed, and, finally, the Pauli–Villars masses are taken to infinity. Calculations done to date[4,5] do not show a great sensitivity to numerical resolution, above a modest threshold, so that the basis sizes used in the matrix problem have been manageable. The largest basis used is approximately 10.5 million states.

2. Yukawa Theory

The light-cone Hamiltonian for Yukawa theory is given by McCartor and Robertson.[8] We work with a single-fermion truncation[6] in which no pair creation or annihilation terms appear and for which an eigensolution is sought only in the one-fermion sector. An analysis of the one-loop fermion self-energy then shows[9,4] that three Pauli–Villars bosons are necessary and that their couplings to the fermion are fixed as functions of their masses by three algebraic conditions. Two of these bosons must be negatively normed. A mass counterterm is also included. The singularity in the instantaneous fermion contribution is canceled by addition of an effective interaction patterned after the contribution of a Z graph.[6]

Renormalization is done by holding fixed the mass of the dressed fermion state and by fixing the value of the expectation value for $:\phi^2(0):$, where ϕ is the boson field operator. The eigenvalue problem is rearranged so that the coefficient of the mass counterterm becomes the eigenvalue. In this form the eigenvalue problem is solved simultaneously with the condition on $\langle :\phi^2(0): \rangle$ by iterating in the value of the bare coupling. This determines the bare mass and bare coupling as functions of the numerical parameters and the regularization parameters. The Fock-state wave functions are also obtained.

3. Numerical Methods and Results

The matrix eigenvalue problem is solved with a variant of the biorthogonal Lanczos method[10] designed specifically for an indefinite metric.[6] This iterative method requires stopping criteria, which we take to be convergence of the eigenvalue and of parts of the boson-fermion wave function, as well. Because the method generates several eigenvalues, we also need criteria for selecting the state of interest. This state is the ground state, but due to the indefinite metric, it is not necessarily the state of lowest mass. The criteria used for selection include the following: a positive norm, a real eigenvalue, absence of nodes in the parallel-helicity boson-fermion wave function, and a relatively large bare-fermion probability. A starting point for the iterations is generated with use of high-order Brillouin–Wigner perturbation theory.

An important check on the calculation is found in the antiparallel boson-fermion wave function, which due to J_z conservation must be in an $L_z = 1$ state. The calculation does not assume this symmetry but instead computes the wave function at all n_x and n_y values. This wave function is found to have the correct symmetry.

Given a method for the computation of Fock-state wave functions, any number of

interesting quantities can be subsequently computed. For example, matrix elements of the fermion current operator yield form factors for the dressed state.[11]

4. Future Work

The techniques described here are applicable to quantum electrodynamics and possibly quantum chromodynamics. In the latter case one would need to use a formulation such as that of Paston *et al.*[12] where an appropriate number of ghost particles and counterterms are inserted, or perhaps a theory with (broken) supersymmetry.

In the short term, a more complete investigation of Yukawa theory can be made. The two-fermion sector of the no-pair version is of interest because one can consider true bound states and scattering states. The full theory is also interesting because pair terms bring additional divergences and renormalization of the boson mass. These cases can be pursued with direct extensions of present methods.

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